Impact of Geometric Phase on Dynamics of Complex-Forming Reactions: H + O$_2$ → OH + O

Junyan Wang, Changjian Xie,* Xixi Hu,* Hua Guo,* and Daiqian Xie*

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ABSTRACT: Reaction dynamics on the ground electronic state might be significantly influenced by conical intersections (CIs) via the geometric phase (GP), as demonstrated for activated reactions (i.e., the H + H$_2$ exchange reaction). However, there have been few investigations of GP effects in complex-forming reactions. Here, we report a full quantum dynamical study of an important reaction in combustion (H + O$_2$ → OH + O), which serves as a proving ground for studying GP effects therein. The results reveal significant differences in reaction probabilities and differential cross sections (DCSs) obtained with and without GP, underscoring its strong impact. However, the GP effects are less pronounced for the reaction integral cross sections, apparently due to the integral of the DCS over the scattering angle. Further analysis indicated that the cross section has roughly the same contributions from the two topologically distinct paths around the CI, namely, the direct and looping paths.
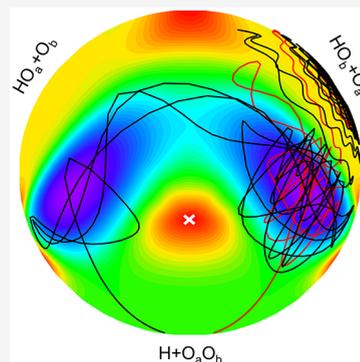
The endothermic H + O$_2$ → OH + O reaction is one of the most important elementary steps in gas-phase hydrocarbon combustion, responsible for chain branching during the oxidation of hydrogen.[1] Due to the existence of a deep HO$_2$ well in the reaction pathway,[2] it also serves as a prototype for complex-forming reactions.[3] For these reasons, this reaction was extensively investigated. Experimentally, its kinetics and dynamics were studied by several groups.[4−13] These experimental studies were complemented by numerous theoretical studies.[14−48] Despite extensive work, a complete understanding of this important process has not been achieved.

A key unresolved issue is that although the reaction nominally proceeds on its electronic ground state (X^{2}A″) potential energy surface (PES), the dynamics might be influenced by the first excited (A^{2}A″) state, because of a common type of electronic degeneracy, namely conical intersections (CIs), between these two states, as shown in Figure 1(a). For an N-dimensional system, a CI is an N-2-dimensional cone-shaped seam of degeneracy.[49,50] For the HO$_2$ system, the branching space is spanned by the H−O$_2$ distance (R) and the Jacobi angle (γ). The two states are coupled near the crossing seams by derivative coupling, which enters the kinetic energy operator as a vector potential in the adiabatic representation.[51] Interestingly, the adiabatic ground state electronic wave function changes its sign along a loop encircling a CI, leading to the so-called geometric phase (GP) in the nuclear wave function,[52] which is related to the Berry's phase.[53] Especially, the wave function acquires a phase (π) each time the system loops around the CI, regardless of the actual path. As a result, two paths passing the CI on different sides might interfere, resulting in a significant impact on

quantum dynamics,[54−58] even when the energy is below the crossing seam. Hence, ignoring the GP in the adiabatic treatment of the dynamics could lead to significant errors.[59,60] The GP effect has recently been observed experimentally for the activated H + HD → D + H$_2$ reaction,[61] which is affected by a high-energy CI, confirming a theoretical prediction made a long time ago.[51] However, the GP has been ignored in all previous quantum scattering investigations of the complex-forming H + O$_2$ reaction, as only the ground electronic state PES was used in such calculations.[21−24,26−32,34−38,41−44,47,48]

Interestingly, earlier quantum dynamics studies of the H + O$_2$ inelastic scattering have revealed significant GP effects.[62−64] Since the complex-forming nature of this reaction differs markedly from that of the direct H + H$_2$ reaction in the dynamics that are strongly influenced by metastable resonances,[3] the impact of GP might also be quite distinct. Already, GP was found to be important for the reverse OH + O reaction at ultracold temperatures.[65−67]

The inclusion of the GP effect in the adiabatic representation by adding the vector potential is numerically difficult, because the derivative coupling and the related diagonal Born−Oppenheimer correction are singular at the crossing seam.[68] Instead, the use of a diabatic representation, in which derivative coupling is removed,[69] is preferred as the
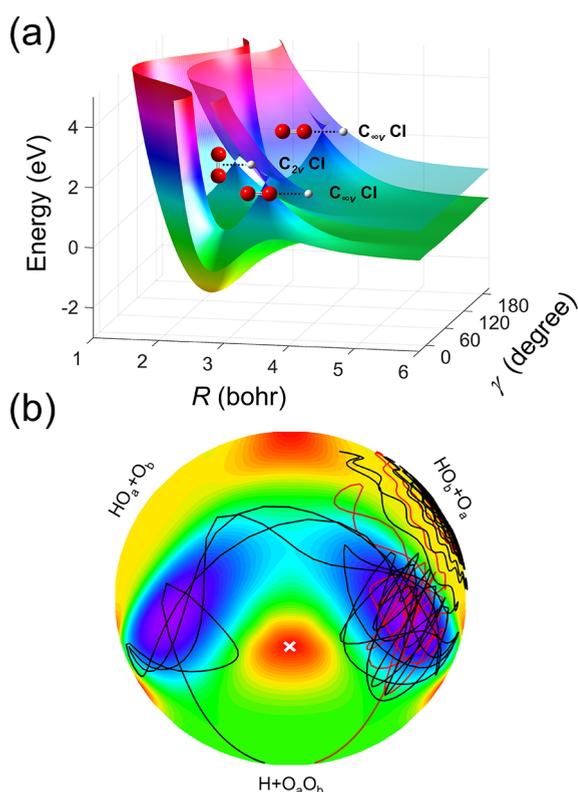
Figure 1. (a) The ground and excited state adiabatic PESs of the HO_2 system in reactant Jacobi coordinates with r_{OO} fixed at 2.28 bohr. Two equivalent linear CIs and a T-shaped CI are clearly seen between the two adiabatic states. (b) Relaxed triangular plot in hyperspherical coordinates of the ground state PES, with the cross for the C_{2v} CI. Exemplary direct (red) and looping (black) trajectories are also shown. (The trajectories were calculated for H + O_2 (v = 0, j = 1) at E_c = 0.70 eV with zero impact parameter.)

GP effect is naturally included with the multistate framework.[70] Unfortunately, rigorous diabatization is not possible for systems with more than two atoms,[71,72] but the residual derivative coupling can be minimized in a so-called quasi-diabatic representation.[73] Diabatization is the determination of the unitary adiabatic-to-diabatic (AtD) transformation that converts the diagonal adiabatic PESs into a diabatic potential energy matrix (DPEM), in which the diagonal and off-diagonal elements are all smooth functions of the nuclear coordinates, thus amenable to analytical representation.[74,75] Recently, we developed a two-state DPEM for the title reaction at the multireference configuration interaction with Davidson correction (MRCI+Q) level with a large basis set.[76] This machine-learned high-fidelity DPEM based on ab initio data is expected to be more accurate than the existing one based on the diatom-in-molecule (DIM) form,[25] thus providing a reliable platform for quantum characterization of the reaction dynamics to unravel the impact of the CIs in this key complex-forming reaction.

Here, we report the first nonadiabatic quantum dynamics study of the H + O_2 (v = 0, j = 1) → OH + O reaction based on our new DPEM, along with adiabatic results for comparison. Such calculations are extremely challenging because of the large basis set needed to cover the vast phase space accessed by the reaction. The total reactive integral cross sections (ICSs) at collision energies (E_c) below 0.80 eV and the differential cross section (DCS) at E_c = 0.70 eV are

obtained. The comparison of the nonadiabatic and adiabatic results provides valuable insight into the role of the GP in reactive scattering dominated by long-lived resonances.

To unravel the influence of the GP on the reactive dynamics, we performed both adiabatic and diabatic calculations using the Chebyshev propagator,[77] with details of our quantum dynamics given in the Supporting Information (SI). Since the adiabatic calculation ignores the GP effect, the results will be denoted below as NGP. Analogously, the diabatic calculations include the GP effect and will be denoted as GP. The wave functions in the diabatic and adiabatic representations are connected with each other by the unitary AtD transformation,[74] which is also defined in the SI.

The total reaction probability of the title reaction as a function of the collision energy is displayed in Figure 2 for
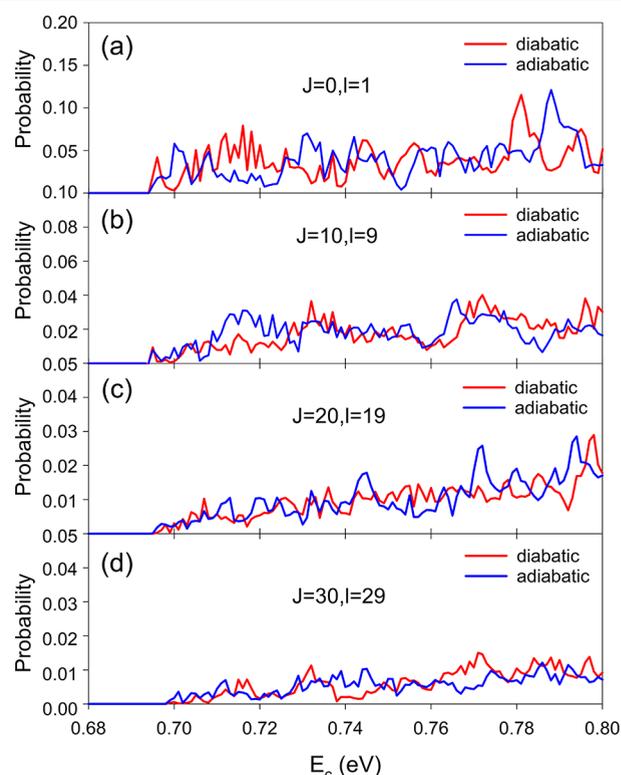
Figure 2. Total reaction probabilities for the reaction H + O_2 (v = 0, j = 1) → O + OH for several partial waves; the red and blue lines show the diabatic and adiabatic results, respectively.

several partial waves. It is clear that the reaction possesses a threshold at 0.695 eV, which well matches the experimental reaction energy (0.72 ± 0.07 eV),[78] underscoring the endoergicity of the reaction. The reaction probabilities strongly oscillate apparently due to metastable resonances supported by the deep HO_2 well. Although the average magnitudes of reaction probabilities are similar, quantitative differences are found between adiabatic and diabatic results even as the collision energies are far below the minimum energy crossing of the CI, which is 1.69 eV above the H + O_2 asymptote.[76] Specifically, the inclusion of the GP in the diabatic results significantly alters the position, width, and amplitude of the resonance peaks in the probabilities. For several energy regions, the GP and NGP results appear to be out of phase. A similar phenomenon was found in probabilities for the H + O_2 inelastic scattering.[62] The reaction probability typically

decreases with the increase of the total angular momentum J due to the increasing centrifugal barrier but remains oscillatory. The GP effect is also remarkable at higher J partial waves.

The GP effect displayed in the reaction probabilities seems to be stronger than several previously studied reactive systems. For the activated H + H$_2$ exchange reaction, the GP effect on probabilities is only significant when the collision energy approaches the energy of the CI.[79,80] For the H + H$_2^+$ → H$_2^+$ + H reaction in its lowest triplet state, which has three equally shallow (0.37 eV) potential wells, the GP effect causes only relatively small changes in total reaction probabilities.[81]

The excitation function, namely, the energy dependence of the ICS, is displayed in Figure 3 up to 0.80 eV. The differences
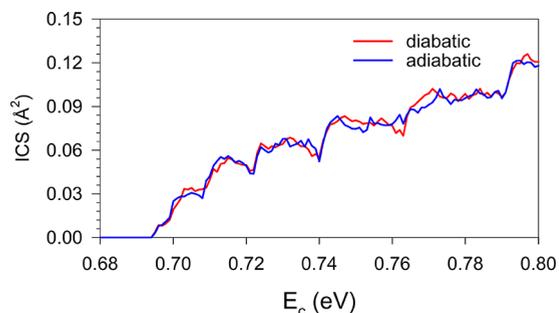
Figure 3. Excitation functions for the reaction H + O$_2$ ($v = 0$, $j = 1$) → O + OH.

between the diabatic and adiabatic ICSs are much less pronounced than the probabilities shown in Figure 2. The sum over partial waves apparently washes away much of the strong oscillations in the reaction probabilities and the GP effects become less pronounced. Unlike the H + H$_2$ exchange reaction in which the GP effects are completely washed away in ICS,[55,80,82,83] some small but distinguishable effects still remain for the title reaction.

The calculated final state resolved DCSs at $E_c = 0.70$ eV are plotted in Figures 4(a) and 4(b). The inclusion of GP clearly alters the angular distribution, but at this energy, the differences are small, except in the forward and backward directions. The relative differences in Figure 4(c) show that the GP effect on the DCS for OH ($v = 0$, $j = 0$) is larger than that for OH ($v = 0$, $j = 1$). In addition, strong oscillations are found in both DCSs at some scattering angles. The GP effect could be understood as the interference between the direct path (labeled as path 1) and looping path (labeled as path 2), which are topologically distinct.[54] In the quantum mechanical framework, results along the two paths could be extracted from linear combinations of the GP and NGP scattering amplitudes (eq 1a).[83,84] The difference between the DCSs obtained from the GP and NGP calculations arises from the interference between two paths, which could be calculated through the sum of the third and fourth terms in eq 1b:

$$f_{\text{path1}}(E) = (f_{\text{NGP}}(E) + f_{\text{GP}}(E))/\sqrt{2}$$

$$f_{\text{path2}}(E) = (f_{\text{NGP}}(E) - f_{\text{GP}}(E))/\sqrt{2} \tag{1a}$$
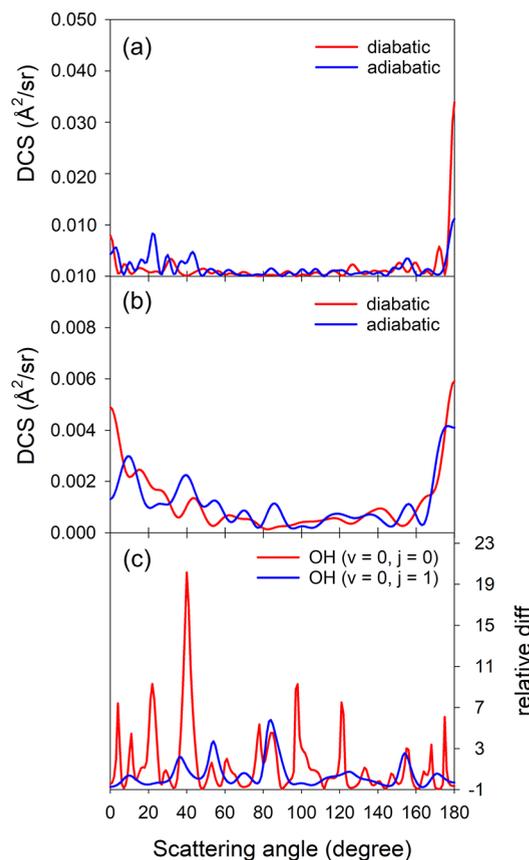
Figure 4. Calculated adiabatic and diabatic DCSs at $E_c = 0.70$ eV for different final states (a and b) and the relative difference between GP and NGP results (c).

$$\sigma_{\text{NGP}} = (|f_{\text{path1}}(E)|^2 + |f_{\text{path2}}(E)|^2 +$$

$$(f_{\text{path1}}(E)f^*_{\text{path2}}(E) + f^*_{\text{path1}}(E)f_{\text{path2}}(E)))/2$$

$$\sigma_{\text{GP}} = (|f_{\text{path1}}(E)|^2 + |f_{\text{path2}}(E)|^2 -$$

$$(f_{\text{path1}}(E)f^*_{\text{path2}}(E) + f^*_{\text{path1}}(E)f_{\text{path2}}(E)))/2 \tag{1b}$$

To illustrate the two topologically distinct paths in the title rection, we perform quasi-classical trajectory (QCT) calculations on the adiabatic ground state PES using VENUS.[85] As shown in Figure 1(b), two typical paths (direct and looping) lead to the same products. In the direct path, the trajectory passes through the HO$_b$O$_a$ well and reaches the HO$_b$ + O$_a$ product asymptote. In the looping path, on the other hand, the trajectory first passes the HO$_a$O$_b$ well and then isomerizes to the HO$_b$O$_a$ well before reaching the HO$_b$ + O$_a$ product asymptote. The isomerization transition state is located at $\gamma = 90°$, $R = 1.74$ bohr, and $r_{\text{OO}} = 2.69$ bohr. The two paths form a complete encirclement of the C$_{2v}$ CI.

The DCSs along the two paths (associated with $|f_{\text{path1}}|^2$, $|f_{\text{path2}}|^2$) and the interference terms (associated with $f_{\text{path1}}(E)f^*_{\text{path2}}(E) + f^*_{\text{path1}}(E)f_{\text{path2}}(E))$ are shown in Figure 5. As mentioned before, the deep potential wells trap the wave functions for a long time, which leads to comparable strengths for the direct and looping paths.[84] As shown in Figure 5, the DCSs along both paths are oscillatory and largely forward−backward symmetric, consistent with the complex-forming character of the reaction. In addition, the GP effects are found
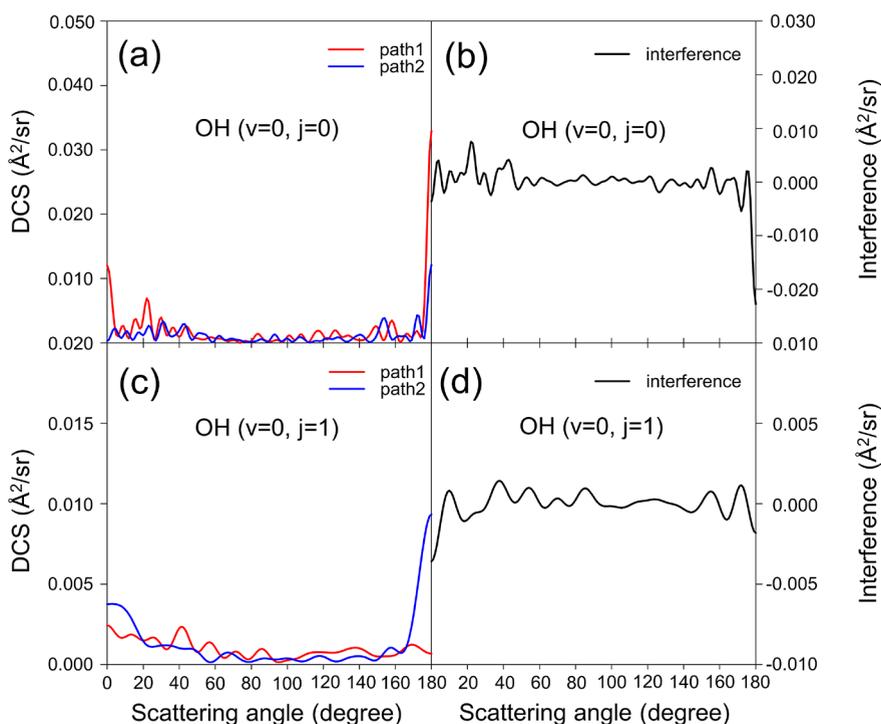
Figure 5. DCSs of the direct (path 1) and looping (path 2) paths at $E_c = 0.70$ eV for different final states (a and c) and the corresponding interference terms (b and d).
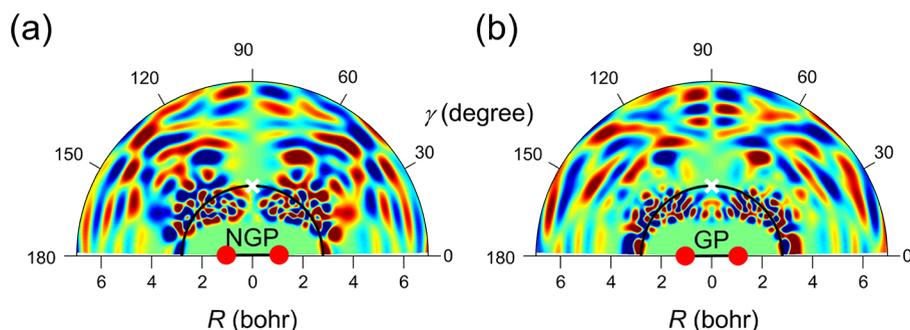
Figure 6. Ground state scattering wave function ($J = 0$) at $E_c = 0.70$ eV with r_{OO} (illustrated as the distance between the red balls) fixed at 2.50 bohr, obtained from adiabatic (a) and diabatic (b) calculations. The cross denotes the location of the C_{2v} CI, and the black circle passing through the CI defines the two regions with small and large R.

to be pronounced at most scattering angles, as shown in Figure 4 for both the $j = 0$ and $j = 1$ products, suggesting the strong interferences between the two paths along the scattering angles. The GP will affect the positions and widths of resonances as featured by the differences in GP and NGP reaction probabilities, thus leading to a pronounced GP effect in DCS. However, the integration of the DCS over the scattering angle washes away much of the GP effect.

The ICSs for path 1 and path 2 at a collision energy of 0.70 eV were calculated. Since the scattering calculations for extracting the S-matrix are quite challenging due to the large size of grids and very long Chebyshev propagation, we only computed the S-matrix at this energy. It is found that the ICS of the direct path (path 1) 0.0242 Å^2 is slightly larger than that (0.0184 Å^2) of the loop path (path 2), suggesting the competition between the two paths.

To obtain further mechanistic insights into the GP effect in the reaction, we show in Figure 6 the ground state scattering wave function at $E_c = 0.70$ eV obtained from both the diabatic

and adiabatic calculations. The former was obtained from the diabatic wave functions via the AtD transformation to facilitate the comparison. The existence of a deep well supports numerous resonance states, as shown in Figure 2, which produce many nodes in the scattering wave function, as shown in Figure 6. Since the branching space of the C_{2v} CI is spanned by R and γ, the wave functions are plotted in these two coordinates. One of the unique signatures of GP is the wave function that encircles the CI changes its sign.[54] This is reflected here by a change of the permutational symmetry when passing through the CI from the small to large R regions.[86] As shown in Figure 6(b), the GP scattering wave function obtained from the diabatic representation is symmetric with respect to the exchange of the two O nuclei in the small R region, but it becomes antisymmetric outside the CI with a node at $\gamma = 90°$. This illustrates the interference between the two paths on opposite sides of the CI.[58] However, this interference feature is absent in the NGP wave function, as shown in Figure 6(a).

To summarize, we investigate in this work the impact of GP on the quantum state resolved H + O$_2$ reactive scattering by comparing adiabatic and diabatic calculations using the same DPEM constructed recently based on high-level ab initio data. Our results suggest that the inclusion of the GP quantitatively changes the reaction probabilities for all partial waves. However, these differences are largely washed out in the ICS, after the DCS is integrated over the scattering angle. Detailed analysis found that the two topologically distinct paths around the CI, i.e., the direct and looping paths, make comparable contributions to the cross sections, and the interference between the two paths leads to a significant impact on the differential cross sections. These insights advance our understanding of GP beyond directly activated reactions.

ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acs.jpclett.4c00789.

> Details of the quantum dynamics methods and parameters used in the calculations (PDF)

AUTHOR INFORMATION

Corresponding Authors

Changjian Xie − *Institute of Modern Physics, Shaanxi Key Laboratory for Theoretical Physics Frontiers, Northwest University, Xi'an, Shaanxi 710127, China;* ◉ orcid.org/0000-0002-2062-430X; Email: chjxie@nwu.edu.cn

Xixi Hu − *Kuang Yaming Honors School, Institute for Brain Sciences, Jiangsu Key Laboratory of Vehicle Emissions Control, Center of Modern Analysis, Nanjing University, Nanjing, Jiangsu 210023, China; Hefei National Laboratory, Hefei, Anhui 230088, China;* ◉ orcid.org/0000-0003-1530-3015; Email: xxhu@nju.edu.cn

Hua Guo − *Department of Chemistry and Chemical Biology, Center for Computational Chemistry, University of New Mexico, Albuquerque, New Mexico 87131, United States;* ◉ orcid.org/0000-0001-9901-053X; Email: hguo@unm.edu

Daiqian Xie − *Institute of Theoretical and Computational Chemistry, Key Laboratory of Mesoscopic Chemistry, School of Chemistry and Chemical Engineering, Nanjing University, Nanjing, Jiangsu 210023, China; Hefei National Laboratory, Hefei, Anhui 230088, China;* ◉ orcid.org/0000-0001-7185-7085; Email: dqxie@nju.edu.cn

Author

Junyan Wang − *Institute of Theoretical and Computational Chemistry, Key Laboratory of Mesoscopic Chemistry, School of Chemistry and Chemical Engineering, Nanjing University, Nanjing, Jiangsu 210023, China*

Complete contact information is available at:
https://pubs.acs.org/10.1021/acs.jpclett.4c00789

Notes

The authors declare no competing financial interest.

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